Abstract—One of most important tasks or key steps in the designing of an EEG-based BCI system is the optimization of spatio-temporal filters for each subject due to the poor spatial resolution of the EEG recordings, as well as the topographical arrangement and frequency specificity of brain activities. A highly popular technique for the optimization of spatial filters is Common Spatial Pattern (CSP). To address the problem of selecting spectral filtering bands, Filter Bank Common Spatial Pattern (FBCSP) was recently proposed, which improves the performance of CSP significantly. This paper provides a deep insight to the CSP algorithm and proposes a novel method termed Time-Constrained Filter Bank Common Spatial Pattern (TFBCSP). TFBCSP eludes the problem of selecting subject-dependent frequency bands by adopting the filter bank approach as in FBCSP and exploits the short-duration nature of ERD/ERS by imposing a time constraint on the EEG samples whose variance is maximized/minimized. Favorable results were obtained by the proposed method on dataset IVa from BCI Competition III.

I. INTRODUCTION

A Brain-Computer Interface (BCI) is an emerging technology that provides a direct communication pathway between a human and an external device. In James Cameron’s epic film ‘Avatar’, a humanoid avatar is controlled by the mind of a paraplegic marine soldier. BCI systems can do amazing jobs like mind-reading, direct control of external devices by human mind in science fictions and films. In real life, the modern BCI is far more crude, confronted with many challenging problems. With the surge of interest among researchers from neuroscience, engineering, and signal processing, there has been fast development for BCI systems in terms of algorithms and applications. Among the various BCI systems, one of the most important BCI systems is the electroencephalogram (EEG)-based motor imagery BCI. In EEG-based motor imagery BCI, EEG signals measured from the scalp during the mental imagination of movement is translated into the command to an external device. The main advantages of EEG are non-invasiveness, and low-cost. However, the designing of an EEG-based BCI system in motor imagery problems is quite complex, given the high variability of the EEG signals for different subjects, target events, and mental states.

Multichannel EEG recordings give a rather blurred image of brain activities because EEG electrodes are separated from current sources in the brain by cerebrospinal fluid (CSF), the skull, and the scalp. Neurophysiological research has shown that macroscopic brain activities are often characterized by a desynchronization/synchronization in certain frequency bands located over various cortical areas, which are termed event-related desynchronization/synchronization (ERD/ERS) [8, 9]. For example, motor activities, both actual and imagined, can often cause an attenuation of the µ-rhythm. Because of the poor spatial resolution of the EEG recordings, as well as the topographical location and frequency specificity of brain activities, one of most important tasks or key steps in the designing of an EEG-based BCI system is the optimization of spatio-temporal filters for each subject. Among the myriad of signal processing methods for EEG-based BCIs, a highly successful spatial filtering method is the Common Spatial Pattern (CSP) algorithm [10], as evidenced by BCI Competition II and III [2, 7]. CSP finds spatial filters that maximize the variance of the spatially filtered signal under one condition while minimizing it for the other condition. Instead of being a black-box process, CSP on band-pass filtered EEG signals corresponds to neurophysiological understandings of ERD/ERS. Since variance of band-pass filtered signals is equal to band-power, the maximization/minimization of variance is equal to maximization/minimization of band-power of the filtered signal. CSP analysis performed on band-pass filtered EEG signals effectively discriminates the mental states corresponding to ERD/ERS. The success of CSP thereby greatly depends on a proper selection of subject-dependent frequency bands and time periods for which CSP is applied on.

Various extensions of the original CSP have been proposed to enhance its performance. Instead of manual selection of frequency bands for the successful application of CSP, simple frequency filters are determined for each channel simultaneously with the spatial filters in Common Spatio-Spectral Pattern (CSSP) [4]. A temporal FIR filter, which can be of higher complexity than the simple filters in CSSP, is simultaneously optimized with the spatial filters in Common Sparse Spectral Spatial Pattern (CSSSP) [3]. The Filter Bank Common Spatial Pattern (FBCSP) [1] proposed the use of filter bank to avoid the problem of selecting spectral filtering bands for CSP. To exploit the spatial relationship between the electrodes, a spatially regularized CSP was proposed in [6]. An extension from two-class to multi-class by joint approximate diagonalization was proposed in [5].

In this paper, a novel machine learning algorithm termed
Time-Constrained Filter Bank Common Spatial Pattern (TFBCSP) is proposed for processing EEG signals in motor imagery BCI applications. TFBCSP is proposed to address the two important problems for CSP: appropriate selection of frequency bands and time period. The proposed TFBCSP adopts the filter bank approach of FBCSP to avoid the manual selection of frequency bands and imposes a time constraint on the EEG samples whose variance is maximized/minimized. In the following sections, the TFBCSP method and the classification results are presented.

II. PROPOSED METHOD: TFBCSP

A. Problem definition

The task of EEG-based motor imagery BCI is to infer the imagined motions (motor imageries) based on the EEG data measured from the scalp. As the EEG electrodes are separated from current source in the brain by CSF, the skull, and the scalp, the relationship of the EEG data $X \in \mathbb{R}^{C \times N}$ ($C$ is the number of channels and $N$ is the number of time points in a single trial) captured from the scalp and the source signals $S \in \mathbb{R}^{C \times N}$ in the brain can be described as $X = f(S)$ where $f(\cdot)$ is the transfer function of the head. Because the volume conduction of the head can be approximated as a linear transformation [11], $f(\cdot)$ is a linear function and

$$X = AS$$

(1)

where $A$ is the source mixing matrix of size $C \times C$. Note that although there are $C$ sources ($C$ rows) assumed in (1), the number of sources of interest, which is associated with the mental activities of interest, is usually less than $C$ and equal to the number of classes (mental tasks). The other sources are associated with background mental activities or noises in the brain signal. It is beneficial to find a de-mixing matrix $W \in \mathbb{R}^{C \times C}$ such that sources related to the mental activities of interest can be estimated:

$$S = WX.$$  

(2)

Each row of $W$ is referred to as a spatial filter, while each column of $A$ is referred to as a spatial pattern. It is easy to see that $W = A^{-1}$. The determination of $W$ by CSP on band-pass filtered EEG signals within a certain time period (post-stimulus period) corresponds to the neurophysiological understanding of ERD/ERS.

B. ERD/ERS, CSP, and FBCSP

Neurophysiological studies show that processing of motor commands or somatosensory stimuli causes an changes in the activity of local neurons, which in turn results in an attenuation or increase of rhythmic activity called ERP or ERS respectively [8, 9]. Three important characteristics of ERD/ERS should be born in mind when we try to exploit ERD/ERS for the classification of EEG signals: frequency bands, spatial location, and time period (the post-stimulus period).

CSP finds spatial filters $W$ that maximize the variance of the filtered signals from one class while minimizing it for the other class. Applied on band-pass filtered signal, CSP maximizes the power ratio of the two classes within that particular frequency band. This contrast in band power corresponds to neurophysiological understandings of ERD/ERS and could be used to discriminate the two classes.

Let $\Sigma^{(1)}, \Sigma^{(2)} \in \mathbb{R}^{C \times C}$ be the estimates of the spatial covariance matrices of the band-pass filtered EEG signal of the two classes (e.g., left hand imagination and right hand imagination):

$$\Sigma^{(c)} = \frac{1}{|D_c|} \sum_{i \in D_c} \Sigma_i,$$

(3)

where $D_c$ is the set of trials belonging to class $c$ and $|D_c|$ is the size of the set. CSP finds the transformation matrix $W$ that simultaneously diagonalizes the two covariances matrices:

$$W\Sigma^{(c)}W^T = \Lambda^{(c)},$$

(4)

This simultaneous diagonalization can be simply achieved by solving the generalized eigenvalue problem:

$$\Sigma^{(1)}w = \lambda \Sigma^{(2)}w.$$  

(5)

If the eigenvectors $w$ are scaled to satisfy $\Lambda^{(1)} + \Lambda^{(2)} = I$ ($I$ is the identity matrix), i.e., $\lambda^{(1)} + \lambda^{(2)} = 1, \text{for } i = 1, \ldots, C$, a large value of $\lambda^{(1)}$ close to one (which means $\lambda^{(2)}$ close to zero) indicates that the corresponding spatial filter yields signals of class 1 having large power, while signals of class 2 having small power. The correspondence of CSP with ERD/ERS and also the success of CSP for BCI applications entails the employment of a spectral filter to extract information from the relevant frequency band and a spatial filter to contrast the power of signals of the two classes.

The problem of selecting subject-dependent frequency bands is addressed by FBCSP, which uses a filter bank to extract components of different frequencies and then applies CSP to find optimal spatial filters for each frequency band. The detail of FBCSP is given in [1].

C. Time-Constrained FBCSP

The problem of manual selection of frequency bands is addressed by FBCSP. However, there still remains the problem of selection of time period. In the following, we first give a deep insight of CSP by a reformulation of its criterion function. Based on this insight, we imposes a time constraint on the EEG samples by a new definition of the spatial covariance matrices of the two classes. With the time constraint, the spatial covariance matrices measure only the variation of spatial patterns that are within a short-time interval, which corresponds to the short-duration of ERD/ERS.

The spatial covariance matrices $\Sigma^{(c)}$ defined in (3) can be
transformed as:

\[ \Sigma_i^{(c)} = \frac{1}{N} X_i X_i^T = \frac{1}{N} \sum_{j=1}^{N} x_i(j) x_i(j)^T \]

\[ = \frac{1}{N} \sum_{j=1}^{N} \left( \frac{1}{N} \sum_{k=1}^{N} x_i(j) x_i(j)^T \right) \]

\[ = \frac{1}{2N^2} \left\{ \sum_{j=1}^{N} \sum_{k=1}^{N} x_i(j) x_i(j)^T + \sum_{j=1}^{N} \sum_{k=1}^{N} x_i(k) x_i(k)^T \right\} \]

\[ = \frac{1}{2N^2} \left\{ \sum_{j=1}^{N} \sum_{k=1}^{N} x_i(j) x_i(j)^T + \sum_{j=1}^{N} \sum_{k=1}^{N} x_i(k) x_i(k)^T \right\} \]

\[ - \sum_{j=1}^{N} \sum_{k=1}^{N} x_i(j) x_i(k)^T - \sum_{j=1}^{N} \sum_{k=1}^{N} x_i(k) x_i(j)^T \right\} \]

\[ = \frac{1}{2N^2} \sum_{j=1}^{N} \sum_{k=1}^{N} (x_i(j) - x_i(k))(x_i(j) - x_i(k))^T, \] (6)

where \( x_i(j) \), denoting the \( j \)th column of the \( i \)th trial \( X_i \), is the EEG data recorded at time instance \( j \) in the \( i \)th trial.

The spatial covariance matrix for a trial shown in (6) is formulated in a pairwise way. This pairwise definition indicates that the spatial covariance matrix \( \Sigma_i^{(c)} \) measures the mean differences between all pairs of spatial patterns in a single trial, i.e., \( x_i(j) \) and \( x_i(k) \) for \( j, k = 1, \cdots, N \). From this insight, we can get another way of interpreting CSP, that is, it maximizes/minimizes the mean variation between all pairs of spatial patterns. However, we need to only consider variation of nearby samples due to the short-duration of ERD/ERS. Variation between spatial patterns that are separated by a large time interval may be mainly due to non-stationarity nature of brain signals.

The time-constrained spatial covariance matrices for each trial is defined as:

\[ S_i^{(c)} = \frac{1}{2N^2} \sum_{j=1}^{N} \sum_{k=1}^{N} \eta(j-k) S_{j,k} \] (7)

where \( S_{j,k} = (x_i(j) - x_i(k))(x_i(j) - x_i(k))^T \). The symbol \( S \) is used instead of \( \Sigma \) to denote the new time-constrained covariance matrix. \( \eta \) is the ‘time-constraint’ function that limits the calculation of the new covariance matrices \( S_i^{(c)} \) to pairs of spatial patterns that are within a certain time interval. The time-constraint function \( \eta \) decreases as the time interval between the \( j \)th and \( k \)th spatial patterns increases.

Since we are interested in information within certain frequency bands, e.g., \( \beta \) band, variation of spatial patterns within very short time period can be left out because it is mainly due to high frequency noise. Removing the symmetry of \( x_i(j) - x_i(k) \) and \( x_i(k) - x_i(j) \) in (7), the covariance matrix \( S_i^{(c)} \) in (7) is simplified as:

\[ S_i^{(c)} = \frac{1}{N^2} \sum_{j=1}^{N-\tau} \sum_{k=j+\tau:N} \eta(j-k) S_{j,k} \] (8)

where \( \tau \) is the sampling interval for the calculation of \( S_i^{(c)} \). Variation of spatial patterns that are within the same sampling interval is left out in (8).

In this paper, we adopt a Gaussian-like function for the \( \eta \) function, defined as

\[ \eta(\Delta t) = \begin{cases} \exp \left( -\frac{\Delta t^2}{2\sigma^2} \right) & \text{if } |\Delta t| < 3\sigma \\ 0 & \text{otherwise} \end{cases} \] (9)

where \( \sigma \) determines the length of the time constraint.

Substituting (9) into (8), we have

\[ S_i^{(c)} = \frac{1}{N^2} \sum_{t=1}^{N-\tau} \sum_{d=1}^{d_0} \eta(dt) S_{t,t+d\tau} \] (10)

where \( d_0 \) is the largest integer that satisfies \( d\tau < 3\sigma \) and \( t + d\tau < N \).

The spatial covariance matrices of the two classes estimated by TFBCSP are thus

\[ \Sigma^{(c)} = \frac{1}{N_0} \sum_{i \in D_0} S_i^{(c)}, \quad c \in \{1, 2\}, \] (11)

where \( S_i^{(c)} \) is defined in (10).

The spatial filters \( W \) of TFBCSP are optimized by simultaneous diagonalization of the spatial covariance matrices of the two classes as defined in (11) and can be easily solved by the generalized eigenvalue problem just as (5) of CSP. As for FBCSP in [1], a filter bank is used by TFBCSP to extract optimal spatial filters for each frequency band. Instead of directly selecting a subject-specific time-period, the proposed method addresses this problem by imposing a time constraint in the definition of the spatial covariance matrices.

### III. EXPERIMENTS

The performance of the proposed TFBCSP for motor imagery BCI applications is tested on EEG data set IVa from BCI Competition III [2]. The dataset is collected from 5 subjects (labeled ‘aa’, ‘al’, ‘av’, ‘aw’, and ‘ay’) who performed right hand and right foot motor imagery. The data for each subject comprises 500 trials of EEG measurements from 118 electrodes. We took the 500-2500 msec interval after visual cue of each trial.

We implemented 5 different algorithms to investigate the effectiveness of the proposed TFBCSP. The 5 algorithms are 1) CSP with no frequency filtering, 2) CSP with wide-band filtering (8-30Hz), 3) Time-Constrained CSP without filter bank (denoted as TCSP), 4) FBCSP, and 5) TFBCSP. The purpose of implementing 1) is to show the importance of spectral filtering for CSP. Algorithms 3, 4 and 5 are implemented to show the respective effect of time constraint and filter bank for CSP. As in [1], a zero-phase Chebyshev Type II Infinite Impulse Response (IIR) filter is used in the filter bank. The filter bank extracts 9 components ranging from 4 to 40 Hz with a bandwidth of 4Hz. The sampling interval \( \tau \) and time constraint \( \sigma \) (or \( d_0 \)) are simply set as \( \tau = 0.03 \text{ sec} \times 100 \text{ samples/sec} = 3 \text{ samples} \), and \( \sigma = 0.25 \text{ sec} \times 100 \text{ samples/sec} = 25 \text{ samples} \), since the sampling frequency is 100Hz. A common parameter
that needs to be set for all the 5 algorithms is the number of spatial filters selected from the set of spatial filters \( W \). We selected the first 2 and last 2 spatial filters. The classifier used is Support Vector Machine (SVM).

The classification performance is obtained by \( 10 \times 10 \)-fold cross-validation. The \( 10 \times 10 \)-fold cross-validation procedure randomly divides the dataset into ten equally sized distinct partitions. Each partition is then used for testing, while other partitions are used for training. This results in 10 accuracies or error rates, which are averaged to give the accuracy of \( 10 \times 10 \)-fold cross-validation. The \( 10 \times 10 \)-fold cross-validation procedure is repeated ten times and the accuracies of the ten runs are again averaged.

Table I shows the test accuracies, estimated by \( 10 \times 10 \)-fold cross-validation procedure. Note that TCSP is applied directly on the raw EEG data without any frequency filtering. Comparing the performance of TCSP with CSP on band-pass filtered (8-30Hz) signals, we can observe that the accuracies of TCSP are comparable to those of CSP on band-pass filtered EEG data. However, if we compare TCSP to CSP without any band-pass filtering, the advantage of TCSP is obvious. The use of filter bank for extracting optimal spatial filters for different frequency components further improve the performance of CSP and TCSP. Overall, TFBCSP achieves the best accuracies for subjects ‘al’, ‘av’, ‘aw’, and ‘ay’, as well as the average accuracy over the five subjects.

IV. CONCLUSIONS

This paper proposed a novel machine learning algorithm termed Short-Term Filter Bank Common Spatial Pattern (TFBCSP) for processing EEG data in motor imagery BCI applications. Corresponding to the neurophysiological phenomena of ERD/ERS, the proposed method eludes the problem of selecting frequency bands by a filter bank and exploits the short-duration nature by the formulation of time-constrained spatial covariance matrices. Preliminary results on data from BCI Competition III dataset IVa demonstrated the effectiveness of the proposed TFBCSP.

REFERENCES


